Practice Problems---Bayes’ Rule.

1. Your friend is studying for an exam. Based on your knowledge of your friend, you believe that if they study for the exam, there is an 80% probability they will be able to pass it. On the other hand, if they do not study, there is only a 30% probability they will be able to pass. Your friend is not a particularly industrious student, and you initially believe there is only a 60% probability your friend will study for the exam. A few days later your friend happily proclaims that they passed the exam. Thus, find the probability that they did in fact study for the test with this knowledge in hand.

2. On the TV show Iron Chef, the weekly challenger must choose one of the three Iron Chefs (Michiba, Sakai, and Chen) to battle. Based on the data on Wikipedia’s Iron Chef page, suppose that if the challenger selects Michiba, there is only a 13% probability they will be able to defeat him. For Sakai and Chen, suppose this probability is 18% and 25% respectively. Since the challenger specializes in Japanese cuisine, you believe there is a 50% probability they will challenge Michiba, a 25% probability they will challenge Sakai, and a 25% probability they will challenge Chen.

   a. What is the probability the challenger wins?

   b. You get distracted and then only see the last minute of the show where you see that the challenger has managed to win. Use Bayes’ Rule (and part (a)) to find the probability that the challenger selected Michiba, given that the challenger has in fact won.

3. I give you three apparently identical shopping bags. One bag (“bag #1”) contains 40 red poker chips and 10 white chips. A second bag contains 25 red chips and 25 white chips, and a third contains 10 red chips and 40 red chips. You choose one of the bags at random, draw a chip, and find the chip was red. What is the probability you drew from bag #1?

4. On his TV show, Gordon Ramsay gives advice (usually in the form of suggestions about the staff and menu) to the owners of failing restaurants. He then revisits each of the restaurants one year later to see if they are still open, and talks to the employees. Ramsay finds that among the restaurants that are still open, 95% implemented his advice, and among the restaurants that closed, only 50% implemented his advice. Further, most of the restaurants visited were in pretty bad shape financially, so assume only 40% would have been able to survive another year.

Thus, use Bayes’ Rule to find P(O|A), the probability a restaurant Ramsay visits remains open for another year, given that they implement his advice.
See the comic above. The machine is able to correctly detect whether or not the sun has gone nova. It then rolls two six-sided dice, and if both come up 6, it lies and if not then it tells the truth.

Suppose you believe that there is a probability of .0001 that the sun will have gone nova at the moment when you ask the machine. Then, the machine tells you that yes the sun has gone nova.

Thus, use Bayes’ Rule to find the probability

\[ P(N|Y), \text{ the probability the sun has in fact gone nova (N), given that the machine says that yes it has (Y).} \]

Note: there are two ways the machine would say the sun has gone nova:

(1) it has not in fact gone nova, and the machine lies (meaning the dice roll was two sixes), or

(2) it has in fact gone nova, and the machine tells the truth (meaning the dice roll was anything other than two sixes).
6. You are a juror in a criminal trial, where the defendant is accused of robbing a bank. Testimony is offered that someone was seen fleeing the bank in a red car, and the DA points out that the defendant does in fact drive a red car. Assume the following:

i. Assuming the defendant is in fact guilty, someone would have been seen in a red car with probability 1 (since he does own a red car, that’s the only car he owns, he is a self-described loner, and based on your observation he seems dumb enough to potentially use his own car as the getaway car).

ii. Assuming the defendant is in fact not guilty, there is a 14% probability that the person seen fleeing the bank would have been driving a red car (perhaps 14% of the cars registered in the area are red).

iii. Prior to hearing this evidence, you believed there was a 25% probability the defendant was guilty based on other testimony.

   a. Use Bayes’ Rule to find \( P(G|E) \), the probability the defendant is guilty given the evidence \( E \) that someone was seen leaving the bank in a red car.

   b. Repeat part (a) if you believed the ‘prior probability’ of the defendant’s guilt was only 1% (instead of 25%).

7. A witness testifying in a hit-and-run case indicates that a green cab hit the plaintiff in the case. You are aware that 85% of the cabs in the city are blue and the other 15% are green (and you believe the witness is credible and competent enough to know a cab when they see one!). However, the defense attorney conducts an experiment recreating the conditions at the scene, and the witness (when shown green and blue cabs) is only able to correctly differentiate them 80% of the time. What is the probability that in fact a green cab (to say nothing about the green cab driven by the defendant!) was involved in the accident? Assume that prior to the witness’ testimony, you would believe there was a 15% probability that a green cab was involved.

   (source: Thinking Fast and Slow by D. Kahneman, p. 161)

8. A statistics student is trying to determine the probability of throwing a total of 7 with two fair six-sided dice. They reason (incorrectly) that since there are 11 possible throws \((2, 3, 4, \ldots, 12)\), this probability is \(1/11\) (when in fact there are 36 (equally likely!) possible throws and six of them \((16, 25, 34, 43, 52, 61)\) yield a 7). To test their hypothesis, they throw two fair dice 3 times and find that a 7 appears all three times.

   Let \( H \) be the student’s flawed hypothesis that a 7 will appear with probability \(1/11\). Let \( E \) be the evidence that that they observed 3 sevens in three throws.

   A friend of the student tries to convince them that \( H \) is incorrect, and suggests a competing hypothesis \(-H\), which is in fact the correct solution to the problem (this probability is \(1/6\)). Unfortunately, as is often the case, suppose the student believes that \( P(H) = .99 \) and \( P(-H) = .01 \). Use Bayes’ Rule to find \( P(H|E) \), the student’s revised probability that \( H \) is true.
9. [Laplace, 1773]. I have a bag containing 10 cards, and each card is either black or white, but you do not know the exact composition of the bag. I draw a single card from the bag and it is white. How many of the cards in the bag are white? Assume that from your point of view, it is initially equally likely that the bag contains 0, 1, 2, ..., or 10 white cards.

10. A family tree tracing a certain genetic disorder is below. The condition in question is recessive (meaning aa’s have the condition, and Aa’s are carriers).

The father knows he is not aa (it’s a condition where it would be obvious if that was the case). Thus, the father’s prior probability of being AA is 1/3 and his prior probability of being Aa is 2/3. Use Bayes’ Rule to find the probability that the father is Aa, given the evidence that all four of his children have also been found to be AA.